

Letters

Comments on “Study of Whispering Gallery Modes in Double Disk Sapphire Resonators”

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In the above paper¹ is presented a simplified, but quite accurate, method for the calculation of the resonant frequencies of whispering gallery modes in dielectric resonators. We have compared the result obtained by Peng with the results of the full-wave radial mode-matching method [1], finding very good agreement. However, a part of the paper describing a model of coupled resonators and, particularly its solution were, in our opinion, faulty.

First of all, the solution of a set of two equations (5), even simplified, cannot be like the solution given by (8). Moreover, the analytical solution of the set is simple and is given by

$$\begin{aligned} k &= \sqrt{1 - \frac{(\omega_1^2 + \omega_2^2)^2}{\omega_1^2 \omega_2^2} \frac{\omega_+^2 \omega_-^2}{(\omega_+^2 + \omega_-^2)^2}} \\ &= \sqrt{1 - \frac{(f_1^2 + f_2^2)^2}{f_1^2 f_2^2} \frac{f_+^2 f_-^2}{(f_+^2 + f_-^2)^2}}. \end{aligned} \quad (1)$$

The expression for the coupling coefficient (8) given by Peng is valid if and only if the resonant frequencies of two separated resonators are the same as (the resonators can be different) what has been already shown in [2] and [3].

When equation (8) of Peng’s paper¹ is applied to the system of two different coupled resonators, then the initial difference of resonant frequencies is misinterpreted as a stronger coupling coefficient.

The second problem in modeling by Peng is the application of the inductive coupling model to the coupled resonators operating in the whispering gallery modes TM₇₁₀ and TM₇₁₁. In the inductive coupling model, the resonant frequency for the even mode is higher than for the odd mode. In a case of symmetrical coupled resonators, the even mode can be found by placing a metal (electric) wall in the symmetry plane between resonators, while the odd mode can be obtained with a magnetic wall situated in the symmetry plane. The TM₇₁₁ and TM₇₁₀ modes behave in a different way. For the metal wall in the symmetry plane, one can compute the resonant frequency of the TM₇₁₀ mode, which is lower than that of the TM₇₁₁ mode. The resonant frequency of the TM₇₁₁ mode can be found by placing the magnetic wall in the symmetry plane. From this, it is clear that magnetic coupling cannot describe a case of the coupled resonators operating in the whispering gallery modes TM₇₁₀ and TM₇₁₁. The proper model in this case should have an electric type of coupling. In the models of electric coupling [4], the resonant frequency for the electric wall in the symmetry plane is lower than that for the magnetic wall in the symmetry plane. The most appropriate model

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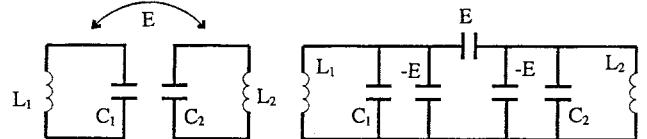
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¹H. Peng, *IEEE Trans. Microwave Theory Tech.*, vol. 44, no. 6, pp. 848–853, June 1996.



$$f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}} \quad f_2 = \frac{1}{2\pi\sqrt{L_2 C_2}} \quad k = \frac{E}{\sqrt{C_1 C_2}}$$

Fig. 1. Series electric coupling model.

is shown in Fig. 1. It is called the series electric coupling model. What is interesting is that the same equations describe the series electric coupling model and parallel magnetic coupling model (Fig. 4 in the above paper¹) with only one, albeit distinct difference. For the electric coupling, f_+ is lower than f_- , thus, in the (5), they should be interchanged:

$$\omega_{\pm}^2 = \frac{\omega_1^2 + \omega_2^2 \mp \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4k^2\omega_1^2\omega_2^2}}{2(1 - k^2)}. \quad (2)$$

This has no effect on a value of the coupling coefficient that can still be calculated using (1) as follows. It is worth mentioning that there are six basic models of reactance coupling [4], i.e., three models of series coupling and three models of parallel coupling. These models also comprise mixed couplings [4], [5].

Unfortunately, as with any models, in some cases they are too simple. The reality is sometime more complicated, as in a case of strong coupling between dielectric resonators operating in the coupled modes EH₁₁₁ and EH₁₁₂ [6], [7], which is an interesting case because these modes “tune across each other.” By recalling this example, we want to stress that models only describe reality. If reality is different than models, than new models are needed. Moreover, models cannot decide how reality should work. All of this is related to the conclusion drawn by Peng on the basis of parallel magnetic coupling model, that “the coupled modes do not tune across each other.” In our opinion, it goes too far. “The mode transition” shown in Peng’s Fig. 6 is rather a simple crossing of characteristics.

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